ORDRE DES INGÉNIEURS DU QUÉBEC

MAY 2014 SESSION

Open-book examination
Calculators : only authorized models
Duration : 3 hours

14-LO-A1 -- Algorithms and Data Structures

QUESTION 1 [20 POINTS]

Complexity Analysis

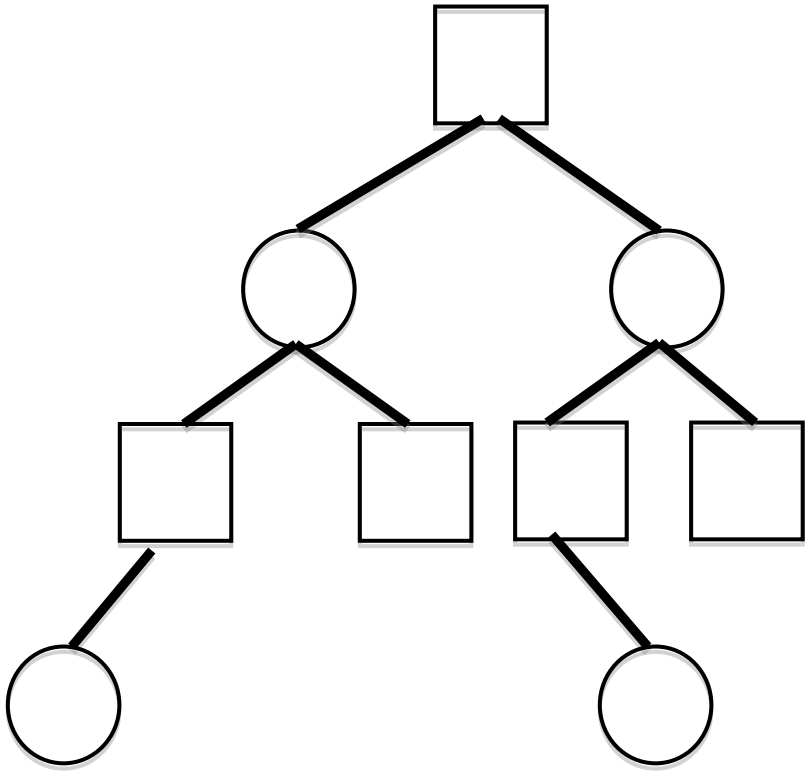
For each of the following algorithms, give the order of growth using the big O notation.

$F_1(n,m)$	$F_2(n,m)$
i ← 1	i ← 1
j ← 1	j ← 1
while i <= m and j <= n	while i <= m or j <= n
i ← i+1	i ← i+1
j ← j+1	j ← j+1
$F_3(n,m)$	**$F_4(n)$**
i ← 1	z ← 0
j ← 1	for x ← 1 to n
while j <= n	for y ← 1 to 100
if i <= m	z ← z + y
i ← i+1	for j ← 1 to n
else	s ← 1
j ← j+1	while s < n
	s ← s * 4

QUESTION 2 [20 POINTS]

Red-Black tree

a) [2 points] Place the numbers 9, 12, 17, 18, 20, 25,26, 28 and 32 in the following red-black tree according to the red-black tree rules. Note that you don't have to insert elements starting from an empty tree, just place them at the correct position in the tree. Also note that black nodes are represented by squares and red nodes by circles.

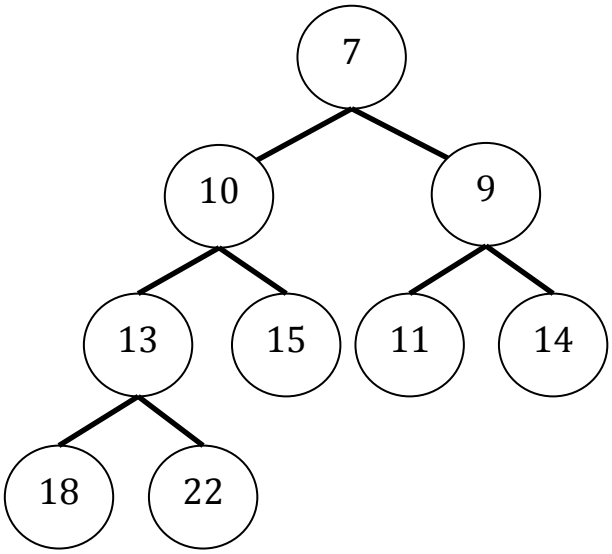


b) [4 points] Insert 14 in the red-black tree obtained upon completion of question (a)
c) [7 points] Insert 27 in the red-black tree obtained upon completion of question (b)
d) [7 points] Insert 10 in the red-black tree obtained upon completion of question (c)

QUESTION 3 [20 POINTS]

Heap

Consider the following heap:

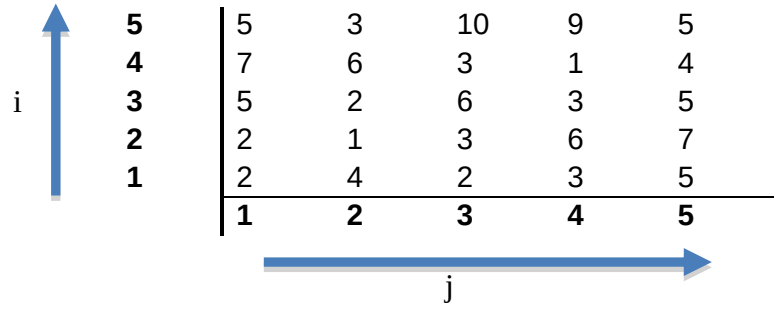


a) (10 points) Insert values 8, 16 and 6 in the heap. (show all steps)
b) (10 points) Extract the two smallest numbers from the heap obtained upon completion of question (a) (Show all steps)

QUESTION 4 [20 POINTS]

Dynamic Programming

Consider a 5x5 discretized environment. A reward is associated to each region of this environment. The function r(i,j) returns a reward for the region at coordinates i,j where i and j are respectively a row and column index. The function r(i,j) is defined as:



You can start from any cell of the first row. You can move 1 step at a time in the 3 possible directions: forward diagonal left, forward and forward diagonal right. For example, if you are at position (1,3), you can go to (2,2), (2,3) or (2,4).

The goal is to find the path that leads to the biggest reward. You must start at row 1 and finish at row 5.

The function q(i,j) defines the best reward that can be obtained at position (i,j).

a) (10 points) Fill in the following table using dynamic programming.

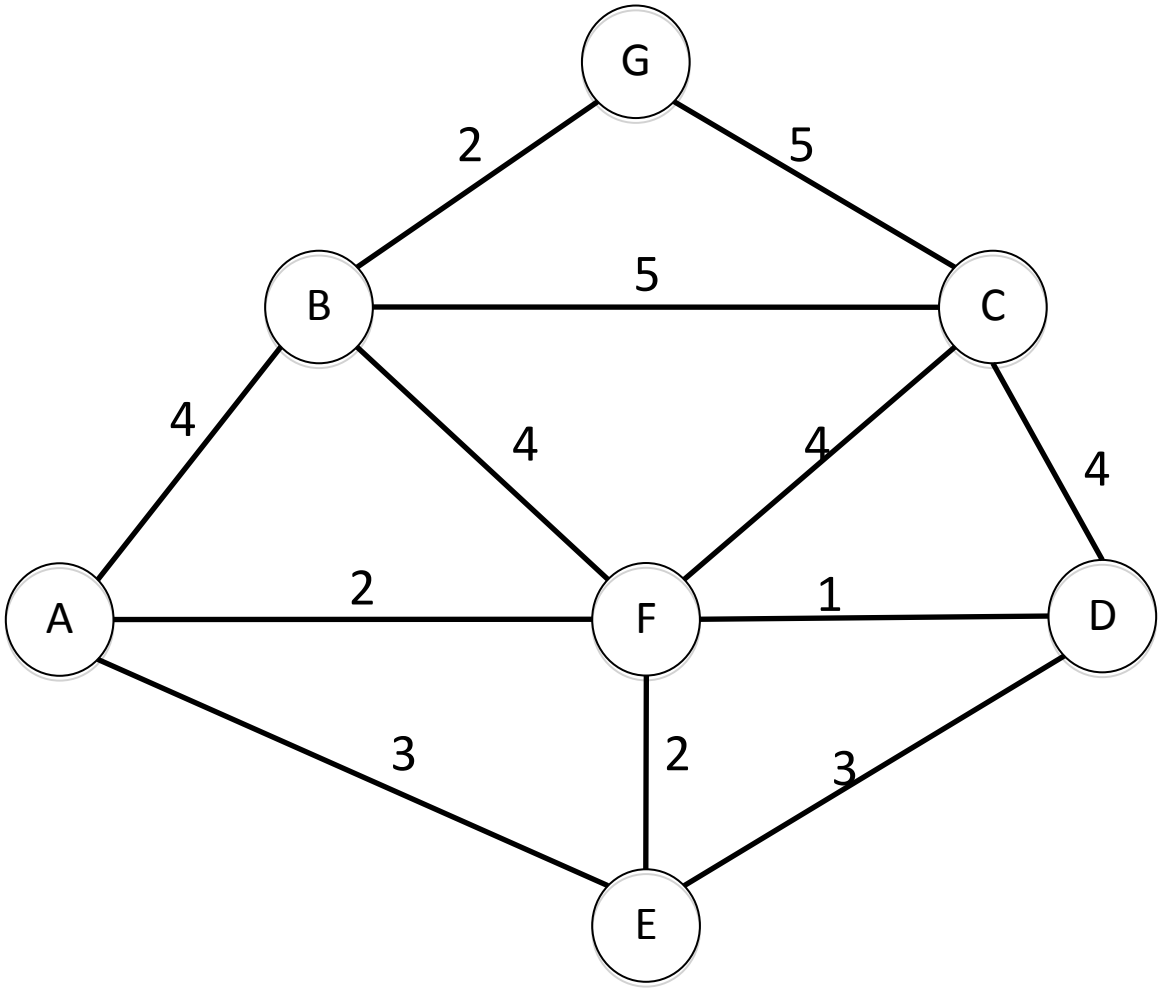
5					
4					
3					
2					
1					
	1	**2**	**3**	**4**	**5**

b) (5 points) Give the recurrence equation.
c) (5 points) Write the algorithm that fills the matrix and that returns the reward.

QUESTION 5 [20 POINTS]

Minimum spanning tree

Consider the following graph:



a) [15 points] Draw the minimum spanning tree associated with this graph.
b) [5 points] What is the total cost of the minimum spanning tree?